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1.) $\displaystyle\int \frac{11-3x}{x^2+2x-3}\,dx$

$$\frac{11-3x}{x^2+2x-3} = \frac{11-3x}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$$

$A(x+3) + B(x-11) = 11-3x$

$Ax + 3A + Bx - 11B = 11-3x$

$A+B = -3$ — ① $\qquad 3A - B = 11$ — ②

$A = -3 - B$ — ③ $\qquad 3(-3-B) - B = 11,\ -9-3B-B = 11$

$-4B = 20 \qquad B = {}^{-20}/_{4} = -5$

$A = -3-(-5) = -3+5 = 2$

$$\therefore \frac{11-3x}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{5}{x+3}$$

$$= \int \frac{2}{x-1}\,dx - \int \frac{5}{x+3}\,dx$$

Let $u = x-1 \qquad v = x+3$

$du/dx = 1 \qquad dv/dx = 1$

$$2\int \frac{du}{u} - 5\int \frac{dv}{v} = 2\ln(u) - 5\ln(v) + c$$

$$= 2\ln(x-1) - 5\ln(x+3) + c$$

2.) $\displaystyle\int \frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}\,dx$

$$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+3}$$

$A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2) = 2x^2-9x-35$

$A(x^2+x-6) + B(x^2+4x+3) + c(x^2-x-2) = 2x^2-9x-35$

$Ax^2 + Ax - 6A + Bx^2 + 4Bx + 3B + Cx^2 - Cx - 2C = 2x^2 - 9x - 35$

$A + B + C = 2$ — (i)

$A + 4B - C = -9$ — (ii)

$-6A + 3B - 2C = -35$ — (iii)

Solve simultaneously ; $A = 4$, $B = -3$, $C = 1$

$$\therefore \frac{2x^2 - 9x - 35}{(x+1)(x-2)(x+3)} = \frac{4}{x+1} - \frac{3}{x-2} + \frac{1}{x+3}$$

$$4\int \frac{1}{x+1}\,dx - 3\int \frac{1}{x-2}\,dx + \int \frac{1}{x+3}\,dx$$

Let $u = x+1$ Let $v = x-2$ Let $w = x+3$

$du/dx = 1$ $dv/dx = 1$ $dw/dx = 1$

$$= 4\int \frac{du}{u} - 3\int \frac{dv}{v} + \int \frac{dw}{w} = 4\ln(u) - 3\ln(v) + \ln(w) + c$$

$$= 4\ln(x+1) - 3\ln(x-2) + \ln(x+3) + c$$

3.) $$\int \frac{1}{x^2 + 121}\,dx$$

Given a right angled triangle ; $\tan\theta = x/11$

$\therefore x = 11\tan\theta$ $dx/d\theta = 11\sec^2\theta$

$$dx = 11\sec^2\theta \cdot d\theta$$

Substituting $x = 11\tan\theta$ and $dx = 11\sec^2\theta \cdot d\theta$

$$\int \frac{11\sec^2\theta \cdot d\theta}{(11\tan\theta)^2 + 121} = \int \frac{11\sec^2\theta \cdot d\theta}{121\tan^2\theta + 121}$$

Recall ; $1 + \tan^2\theta = \sec^2\theta$

$\therefore 121\tan^2\theta + 121 = 121\sec^2\theta$

$$= \int \frac{11\sec^2\theta \cdot d\theta}{121\sec^2\theta} = \int \frac{d\theta}{11} = \frac{1}{11}\int d\theta$$

$$= \frac{1}{11}[\theta] + c$$

, If $\tan\theta = x/11$; $\theta = \tan^{-1}[x/11]$

$$= \frac{1}{11}\tan^{-1}\left[\frac{x}{11}\right] + c$$